957881

11223

II Semester B.A./B.Sc. Degree Examination, September - 2021
MATHEMATICS
(CBCS Scheme (F+R) 2014-15 & Onwards)
Paper : II

Time : 3 Hours　　　　　　　　　　　　　　　　**Maximum Marks : 70**

Instructions to Candidates:

　　　　Answers **ALL** Parts.

PART - A

Answer any **FIVE** questions.　　　　　　　　　　　　　　(5×2=10)

1.　a)　The binary operation $*$ is defined on the set z of integers by
　　　　$a*b = a+b-2 \; \forall a,b \in z$ Find the identity element.

　　b)　Prove that in a group $(G,*), \left(a^{-1} \right)^{-1} = a, \; \forall a \in G$

　　c)　Write the formula for radius of curvature in parametric form.

　　d)　Find the length of the polar subnormal at the point $\theta = \dfrac{\pi}{6}$ for the curve $r = a\cos 2\theta$

　　e)　Find the asymptotes parallel to the co-ordinate axes for the curve $\left(x^2 + a^2 \right) y = bx^2$

　　f)　Write the formula to find the length of an arc of the curve
　　　　$y = f(x)$ from $x = a$ to $x = b$

　　g)　Show that the equation $\left(2xy + 3y \right)dx + \left(x^2 + 3x \right)dy = 0$ is exact.

　　h)　Solve $P^2 - 5P + 6 = 0$ where $P = \dfrac{dy}{dx}$

[P.T.O.

11223

PART - B

Answer **ONE** full question (1×15=15)

2. a) Show that the set of all fourth roots of unity forms an abelian group under multiplication.

b) Prove that a non-empty subset H of a group $(G, *)$ is a subgroup of G, if and only if

   i) $a * b \in H \quad \forall a, b \in H$

   ii) $a^{-1} \in H \quad \forall a \in H$

c) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ be two permutations of order 3 then find (fog) and $\left(f^{-1}og^{-1}\right)$

(OR)

3. a) Let G be the set of all rational number and $*$ be the binary operation on G defined by $a * b = \dfrac{ab}{7} \ \forall a, b \in G$, then prove that $(G, *)$ is an abelian group.

b) Prove that $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12.

c) In a group $(G, *)$, if every element is its own inverse then prove that it is abelian.

PART - C

Answer any **TWO** full questions. (2×15=30)

4. a) With usual notation prove that

   $\tan \phi = r \dfrac{d\theta}{dr}$ for the polar curve $r = f(\theta)$

b) Show that the curves $r = a(1 + \cos\theta)$, $r = b(1 - \cos\theta)$ intersect orthogonally.

c) Show that the evolute of the parabola $y^2 = 4ax$ is $27 ay^2 = 4(x - 2a)^3$

(OR)

5. a) Find the angle of intersection of the curves $r = \dfrac{a}{\theta}$ and $r = a\theta$

b) Derive the formula for radius of curvature in cartesian form.

c) Find the Pedal equation of the curve $r^n = a^n \cdot \cos n\theta$

6. a) Find all the asymptotes of the curve

$$x^3 + 2x^2 y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

   b) Find the area bounded by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$

   c) Find the position and nature of the double points of the curve

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0$$

(OR)

7. a) Find the length of the arc of the curve $y = \log(\sec x)$ from $x = 0$ to $x = \dfrac{\pi}{3}$

   b) Find the envelope of the family of straight lines $y = mx + \dfrac{a}{m}$, where m is the parameter.

   c) Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

PART - D

Answer any **ONE** full question.   (1×15=15)

8. a) Solve $\dfrac{dy}{dx} + \dfrac{3x^2 y}{1 + x^3} = \dfrac{\sin^2 x}{1 + x^3}$

   b) Solve $y = 2px + y^2 p^3$

   c) Verify for exactness and solve

$$(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$$

(OR)

9. a) Solve $\dfrac{dy}{dx} + 2xy = 2e^{-x^2}$

   b) Find the general and singular solution of $y = Px + P^2$

   c) Find the orthogonal trajectory of the family of the curves $y^2 = 4ax$.

_____